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Transport-Theoretic Interpretation of Basic Lagrangians

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We show how one can interpret the Lagrangians defining quantum and scalar electrodynamics as asymptotic descriptions of certain relativistic transport-theoretic Lagrangians. It is suggested that the gauge parameter, regularizations, and renormalizations may have their origins in the way such transport-theoretic Lagrangians are asymptotically approximated. As an alternative to the Higgs mechanism the interaction with Lorentz-invariant ground states of vector bosons is put forward.

1. INTRODUCTION

The Lagrangian density $\mathscr{L}$ is used by contemporary theories of fundamental interactions as the basic description of the physical world in terms of its state Ψ. The Lagrangian formulation most clearly exhibits all global and local symmetries of a theory and provides a common starting point (i) for classical field theories whose equations of motion are given by Hamilton's principle of stationary action

$$\delta I[\Psi] = 0, \qquad I[\Psi] \equiv \int_{t_1}^{t_2} dt \int_{\mathrm{IR}^3} \mathscr{L}(\Psi, \nabla\Psi)\, dV \tag{1.1}$$

and (ii) for relativistic quantum field theories constructed through the path-integral method using Feynman's path integral

$$\int \mathfrak{D}\Psi \exp\{(i/\hbar) I[\Psi]\} \tag{1.2}$$

or through canonical quantization, which both imply the Feynman graph

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expansions. Now, the Lagrangians of quantum field theories have to be amended by the rules governing the gauge parameter, regularizations, renormalizations, and omissions of vacuum–vacuum subdiagrams. So, one can argue that they are actually just asymptotic approximations to some underlying Lagrangian that has been simplified by effective parameters (masses and coupling constants) standing for certain interactions that should, therefore, be omitted from theories based on asymptotic Lagrangians.

Now the question arises of how to arrive at this underlying Lagrangian—provided of course there is one. According to Feynman, sound theories the limits of which are quantum field theories are still an open question of theoretical physics (Mehra, 1994). It was suggested, e.g., by Bjorken and Drell (1965) already 30 years ago that the quantum field theories may not give an adequate description of the physical world at distances smaller than some characteristic length. To obtain an idea how to improve on the conventional Lagrangians of quantum field theories we follow a suggestion by Feynman *et al.* (1965) and reason on the analogy of the kinetic theory of gases, where phenomena smoothed over the time and space variable may be adequately described by the partial differential equations of fluid dynamics in terms of certain fields of the time-space variable $x \in \mathrm{IR}^{1,3}$. On the other hand, a more precise description can be given in terms of the single-particle distribution function that depends on the time-space variable x and four-momentum variable $p \in \mathrm{IR}^{1,3}$, and satisfies an integrodifferential transport equation (Williams, 1971; de Groot *et al.*, 1980). We have shown (Ribarič and Šušteršič, 1995) how one can construct covariant linear integrodifferential transport equations that (a) imply in the strong-scattering asymptote the Klein–Gordon, Dirac, and Proca partial differential equations of quantum field theories, and (b) display faster-than-light effects, though they do not propagate signals faster than light. In what follows we will consider how one can derive from transport-theoretic Lagrangians the conventional Lagrangians of quantum and scalar electrodynamics.

2. TRANSPORT-THEORETIC LAGRANGIANS

Within the proposed transport-theoretic framework, we presume that the state of a physical system is described by a relativistic field $\Psi(x, p)$ of two independent four-vector variables: the time-space variable $x \equiv (ct, \mathbf{r}) \in \mathrm{IR}^{1,3}$ and the four-momentum variable $p \equiv (p^0, \mathbf{p}) \in \mathrm{IR}^{1,3}$; so to say, we assume that physical space has 4 + 4 dimensions. We assume that the Lagrangian density $\mathcal{L}$ of a physical system maps the state $\Psi(x, p)$ and its first-order derivative $\nabla\Psi(x, p)$ with respect to the time-space variable x into a real relativistic-scalar field of $x \in \mathrm{IR}^{1,3}$. We take that the Lagrangian density $\mathcal{L}$ is a sum

$$\mathscr{L}(\Psi, \nabla\Psi) = \mathscr{L}_f + \mathscr{L}_i \tag{2.1}$$

of the free and interaction parts. The free part $\mathscr{L}_f$ completely describes the free states $\Psi_f(x, p)$ that are taken as the primary concept analogous to the concept of free bodies described by Newton's first law. All differences between states $\Psi(x, p)$ and the free states $\Psi_f(x, p)$ are modeled and explained by the interaction part $\mathscr{L}_i$ characteristic of the physical system considered.

2.1. Lagrangian Density $\mathscr{L}_f$ of a Free State

In transport theory a free state $\Psi_f(x, p)$ is assumed to satisfy the covariant equation of motion

$$p\cdot\nabla\Psi_f(x, p) = 0, \qquad \text{with} \quad p\cdot\nabla \equiv p^0 \frac{1}{c}\frac{\partial}{\partial t} + \mathbf{p}\cdot\boldsymbol{\nabla} \tag{2.2}$$

being the covariant substantial derivative (de Groot *et al.*, 1980). This equation is an analog to Newton's first law, since a solution to (2.2) for $p^0 \neq 0$ in terms of its value $\Psi_f(ct_0, \mathbf{r}, p^0, \mathbf{p})$ at $t = t_0$ reads

$$\Psi_f(ct, \mathbf{r}, p^0, \mathbf{p}) = \Psi_f(ct_0, \mathbf{r} - c\mathbf{p}(t - t_0)/p^0, p^0, \mathbf{p}) \tag{2.3}$$

Any state $\Psi(x, p)$ that does not depend on the time-space variable is a free state. A plane wave

$$\Psi_{pw}(x, p) \equiv \Psi_0(p)e^{ip\cdot x} \qquad \text{with} \quad \Psi_0(p) = 0 \quad \text{if} \quad [p^0]^2 - |\mathbf{p}|^2 \neq 0 \tag{2.4}$$

is also a transport-theoretic free state, and it satisfies the same wave equation

$$\nabla\cdot\nabla\Psi_{pw}(x, p) = 0 \tag{2.5}$$

as the free massless fields of quantum field theories. If we interpret the state $\Psi(x, p)$ as describing at x a certain property of pointlike entities characterized by their four-momentum $p \in \mathrm{IR}^{1,3}$, the relation (2.3) tells us that in a free state $\Psi_f(x, p)$ this property is streaming with velocity $c\mathbf{p}/p^0 \in \mathrm{IR}^3$; note that the speed $c|\mathbf{p}/p^0|$ is not bounded. If we take that $\Psi_f(x_1, p_1) \neq 0$ ($= 0$) signifies that such pointlike entities with four-momentum $p = p_1$ are present (absent) at $x = x_1$, then in the case of a spatially localized initial free state, say $\Psi_f(ct_0, \mathbf{r}, p) = 0$ if $|\mathbf{r}| > R_0$, the relation (2.3) points out that in a free state they are spreading with a speed $c|\mathbf{p}_1/p_1^0|$ that may be arbitrarily high. So we could say that in the case of a free state there are pointlike entities with arbitrary four-momenta p that are streaming through the three-dimensional space IR^3. In relativistic kinetic theory one assumes that $p^0 > 0$ and $|\mathbf{p}|^2 < [p^0]^2$, so that the speeds $c|\mathbf{p}/p^0| < c$; here p^0, $|\mathbf{p}|^2 \in \mathrm{IR}$ and so the speeds $c|\mathbf{p}/p^0| \in [0, \infty)$.

To construct a Lagrangian that implies (2.2), we use a mapping $\langle\Psi|\Psi'\rangle_p$ of any two states $\Psi(x, p)$ and $\Psi'(x, p)$ into a complex function of $x \in \mathrm{IR}^{1,3}$. We designate $\langle\vdots|\vdots\rangle_p$ as a scalar product with respect to the independent four-momentum variable p and assume that $\langle\vdots|\vdots\rangle_p$ has the following properties:

(i) It is *local* with respect to the time-space variable x and does not depend explicitly on x, i.e., for any $a \in \mathrm{IR}^{1,3}$,

$$\langle\Psi(x, p)|\Psi'(x, p)\rangle_p\Big|_{x=a} = \langle\Psi(a, p)|\Psi'(a, p)\rangle_p \tag{2.6}$$

$$\langle\Psi(x + a, p)|\Psi'(x + a, p)\rangle_p = \langle\Psi(x, p)|\Psi'(x, p)\rangle_p\Big|_{x\to x+a}$$

(ii) It is *bilinear*, i.e., for any three states $\Psi(x, p)$, $\Psi'(x, p)$, and $\Psi''(x, p)$ and for any two complex constants c', $c'' \in \mathbf{C}$,

$$\langle\Psi|c'\Psi' + c''\Psi''\rangle_p = c'\langle\Psi|\Psi'\rangle_p + c''\langle\Psi|\Psi''\rangle_p$$

$$\langle\Psi|\Psi'\rangle_p = \langle\Psi'|\Psi\rangle_p^* \tag{2.7}$$

with the asterisk denoting complex conjugation (so $\langle\Psi|\Psi\rangle_p$ is real, but not necessarily nonnegative).

(iii) It is a relativistic-scalar field of the time-space variable x, i.e., if the states Ψ and Ψ' transform under Lorentz transformations Λ as, say,

$$\Lambda\Psi \equiv (\Lambda\Psi)\,(\Lambda^{-1}x, \Lambda^{-1}p) \tag{2.8}$$

then for any Λ,

$$\langle\Lambda\Psi|\Lambda\Psi'\rangle_p = \langle\Psi(x, p)|\Psi'(x, p)\rangle_p\Big|_{x\to\Lambda^{-1}x} = \Lambda(\langle\Psi|\Psi'\rangle_p) \tag{2.9}$$

(iv) It is such that

$$\langle\Psi'|\Psi\rangle_p = 0 \quad \forall\, \Psi' \qquad \text{implies} \quad \Psi = 0 \tag{2.10}$$

and

$$\langle\Psi|(a\cdot p)\Psi'\rangle_p = -\langle(a\cdot p)\Psi|\Psi'\rangle_p \qquad \forall\, a \in \mathrm{IR}^{1,3} \tag{2.11}$$

where $a\cdot p \equiv a^0p^0 - \mathbf{a}\cdot\mathbf{p}$.

Using the above properties of the scalar product $\langle\vdots|\vdots\rangle_p$, one can show that the covariant equation of motion (2.2) of a free field is the Euler–Lagrange equation of the Lagrangian density

$$\mathcal{L}_\mathrm{f}(\Psi, \nabla\Psi) \equiv \mathcal{R}\langle\Psi|p\cdot\nabla\Psi\rangle_p = \langle\Psi|p\cdot\nabla\Psi\rangle_p - \tfrac{1}{2}\nabla\cdot\langle\Psi|p\Psi\rangle_p \tag{2.12}$$

where $\mathcal{R}$ denotes the real part. So we take (2.12) as the free part $\mathcal{L}_\mathrm{f}$ of the transport-theoretic Lagrangians (2.1) we are going to consider. Note that $\mathcal{L}_\mathrm{f}(\Psi, \nabla\Psi)$ defined by (2.12) is a relativistic-scalar field of x, i.e., for any Λ,

$$\mathscr{L}_f(\Lambda\Psi, \nabla\Lambda\Psi) = \mathscr{L}_f(\Psi, \nabla\Psi)|_{x\to\Lambda^{-1}x} \tag{2.13}$$

2.2. Interaction Part $\mathscr{L}_i$ of the Lagrangian $\mathscr{L}$

In what follows we assume that the interaction part $\mathscr{L}_i$ of the transport-theoretic Lagrangian $\mathscr{L}$ is such that

$$\begin{aligned}\mathscr{L}_i = \mathscr{L}_i(\Psi) \equiv \mathscr{L}_1(\Psi) + \mathscr{L}_2(\Psi, \Psi) + \mathscr{L}_3(\Psi, \Psi, \Psi) \\ + \mathscr{L}_4(\Psi, \Psi, \Psi, \Psi)\end{aligned} \tag{2.14}$$

in which the operators $\mathscr{L}_1(\Psi_1)$, $\mathscr{L}_2(\Psi_1, \Psi_2)$, $\mathscr{L}_3(\Psi_1, \Psi_2, \Psi_3)$, and $\mathscr{L}_4(\Psi_1, \Psi_2, \Psi_3, \Psi_4)$ have the following properties:

(i) They map states Ψ_1, Ψ_2, Ψ_3, and Ψ_4 into complex relativistic-scalar fields of the time-space variable $x \in \mathrm{IR}^{1,3}$ that do not depend on the four-momentum variable p; e.g., in the case of the trilinear term $\mathscr{L}_3$, for any Λ,

$$\mathscr{L}_3(\Lambda\Psi_1, \Lambda\Psi_2, \Lambda\Psi_3) = \mathscr{L}_3(\Psi_1, \Psi_2, \Psi_3)|_{x\to\Lambda^{-1}x} = \Lambda(\mathscr{L}_3(\Psi_1, \Psi_2, \Psi_3)) \tag{2.15}$$

(ii) $\mathscr{L}_1(\Psi)$, $\mathscr{L}_2(\Psi, \Psi)$, $\mathscr{L}_3(\Psi, \Psi, \Psi)$, and $\mathscr{L}_4(\Psi, \Psi, \Psi, \Psi)$ are real for any Ψ.

(iii) They are totally symmetric with respect to their arguments; e.g., in the case of the quadrilinear term,

$$\mathscr{L}_4(\Psi_1, \Psi_2, \Psi_3, \Psi_4) = \mathscr{L}_4(\Psi_2, \Psi_1, \Psi_3, \Psi_4) = \dots = \mathscr{L}_4(\Psi_4, \Psi_3, \Psi_2, \Psi_1) \tag{2.16}$$

This requirement [that, e.g., $\mathscr{L}_2(\Psi_1, \Psi_2)$ is the symmetric extension of $\mathscr{L}_2(\Psi, \Psi)$] simplifies subsequent results and imposes no additional restrictions on the interaction part $\mathscr{L}_i$.

(iv) They are *linear in each argument*; e.g., in the case of the bilinear interaction term $\mathscr{L}_2$, for any states Ψ_n and for any real constants c_n,

$$\mathscr{L}_2\left(\sum_j c_j\Psi_j, \sum_k c_k\Psi_k\right) = \sum_{j,k} c_j c_k \mathscr{L}_2(\Psi_j, \Psi_k) \tag{2.17}$$

(v) They are *local* with respect to the independent variable x and do not depend explicitly on x, i.e., they satisfy relations analogous to (2.6). Thus, $\mathscr{L}_1$, $\mathscr{L}_2$, $\mathscr{L}_3$, and $\mathscr{L}_4$ represent a kind of contact interactions in a homogeneous time-space $\mathrm{IR}^{1,3}$, since the values of, e.g., $\mathscr{L}_2(\Psi_1, \Psi_2)$ at $x = x_1$ and of $\mathscr{L}_2(\Psi_1', \Psi_2')$ at $x = x_2$ are equal when $\Psi_1(x = x_1, p) = \Psi_1'(x = x_2, p)$ and $\Psi_2(x = x_1, p) = \Psi_2'(x = x_2, p)$.

As a consequence, Lagrangian density $\mathscr{L}$ is form-invariant: (a) under Lorentz transformations,

$$\mathscr{L}(\Lambda\Psi, \nabla\Lambda\Psi) = \mathscr{L}(\Psi(x, p), \nabla\Psi(x, p))\big|_{x\to\Lambda^{-1}x} \qquad \forall\ \Lambda \tag{2.18}$$

and (b) under time-space shifts,

$$\begin{aligned} &\mathscr{L}(\Psi(x + a, p), \nabla\Psi(x + a, p)) \\ &= \mathscr{L}(\Psi(x, p), \nabla\Psi(x, p))\big|_{x\to x+a} \qquad \forall\ a \in \mathrm{IR}^{1,3} \end{aligned} \tag{2.19}$$

2.3. The Euler–Lagrange Equations of the Lagrangian $\mathscr{L}_f(\Psi, \nabla\Psi) + \mathscr{L}_i(\Psi)$

By Hamilton's principle, the Euler–Lagrange equations for state Ψ read

$$\mathrm{EL}(\Psi; \Psi') = 0 \qquad \forall\ \Psi' \tag{2.20}$$

where

$$\begin{aligned} \mathrm{EL}(\Psi; \Psi') \equiv{}& \langle\Psi'|p\cdot\nabla\Psi\rangle_p + \langle p\cdot\nabla\Psi|\Psi'\rangle_p + \mathscr{L}_1(\Psi') + 2\mathscr{L}_2(\Psi', \Psi) \\ &+ 3\mathscr{L}_3(\Psi', \Psi, \Psi) + 4\mathscr{L}_4(\Psi', \Psi, \Psi, \Psi) \end{aligned} \tag{2.21}$$

The zero state $\Psi(x, p) \equiv 0$ is a solution to the Euler–Lagrange equations (2.20) if and only if there is no linear term, i.e., $\mathscr{L}_1 \equiv 0$. If both $\Psi(x, p)$ and $-\Psi(x, p)$ are solutions to (2.20), then, for such a state Ψ, by (2.7) and (2.17),

$$\mathscr{L}_1(\Psi') + 3\mathscr{L}_3(\Psi', \Psi, \Psi) = 0 \qquad \forall\ \Psi' \tag{2.22}$$

The Euler–Lagrange equations (2.20)–(2.21), (2.6), (2.7), and (2.11) imply that any solution Ψ to (2.20) satisfies the covariant continuity equation

$$\begin{aligned} i\nabla\cdot\langle\Psi|p\Psi\rangle_p ={}& \mathscr{L}_1(i\Psi) + 2\mathscr{L}_2(i\Psi, \Psi) + 3\mathscr{L}_3(i\Psi, \Psi, \Psi) \\ &+ 4\mathscr{L}_4(i\Psi, \Psi, \Psi, \Psi) \end{aligned} \tag{2.23}$$

2.4. Shifted Lagrangian

For a given reference state $\Psi_r(x, p)$ we define the difference

$$\Psi_d(x, p) \equiv \Psi(x, p) - \Psi_r(x, p) \tag{2.24}$$

and write the transport-theoretic Lagrangian

$$\mathscr{L}(\Psi, \nabla\Psi) = \mathscr{L}(\Psi_r, \nabla\Psi_r) + \nabla\cdot\mathscr{R}\langle\Psi_r|p\Psi_d\rangle_p + \mathscr{L}_d \tag{2.25}$$

in which the shifted Lagrangian density is

$$\mathcal{L}_d(\Psi_d, \nabla\Psi_d; \Psi_r) \equiv \mathcal{L}_f(\Psi_d, \nabla\Psi_d) + \mathcal{L}_{id}(\Psi_d; \Psi_r) \tag{2.26}$$

with the interaction part

$$\begin{aligned}
\mathcal{L}_{id}(\Psi_d; \Psi_r) &\equiv \mathcal{L}_{1d} + \mathcal{L}_{2d} + \mathcal{L}_{3d} + \mathcal{L}_4(\Psi_d, \Psi_d, \Psi_d, \Psi_d) \\
\mathcal{L}_{1d}(\Psi_d; \Psi_r) &\equiv \mathrm{EL}(\Psi_r; \Psi_d) \\
\mathcal{L}_{2d}(\Psi_d, \Psi_d; \Psi_r) &\equiv \mathcal{L}_2(\Psi_d, \Psi_d) + 3\mathcal{L}_3(\Psi_r, \Psi_d, \Psi_d) \\
&\quad + 6\mathcal{L}_4(\Psi_r, \Psi_r, \Psi_d, \Psi_d) \\
\mathcal{L}_{3d}(\Psi_d, \Psi_d, \Psi_d; \Psi_r) &\equiv \mathcal{L}_3(\Psi_d, \Psi_d, \Psi_d) + 4\mathcal{L}_4(\Psi_r, \Psi_d, \Psi_d, \Psi_d)
\end{aligned} \tag{2.27}$$

Here the state $\Psi_d(x, p)$ is the independent dynamical variable of the shifted Lagrangian $\mathcal{L}_d(\Psi_d, \nabla\Psi_d; \Psi_r)$ and $\Psi_r(x, p)$ is a given reference state, i.e., a kind of parameter. The shifted and original Lagrangians $\mathcal{L}_d$ and $\mathcal{L}$ are physically equivalent. In particular, if $\Psi_d(x, p)$ is a solution to the Euler–Lagrange equations of the shifted Lagrangian $\mathcal{L}_d$, then the sum $\Psi_r + \Psi_d$ is a solution to the original Euler–Lagrange equations (2.20).

The shifted Lagrangian $\mathcal{L}_d$ has, in particular, the following properties: (a) The free part of $\mathcal{L}_d$ is the same as in the original Lagrangian $\mathcal{L}$. (b) The interaction part $\mathcal{L}_{id}$ has no linear term, i.e., $\mathcal{L}_{1d} \equiv 0$, if $\Psi_r(x, p)$ is a solution to the original Euler–Lagrange equations (2.20). (c) Even if the original Lagrangian $\mathcal{L}$ did not have the linear and bilinear terms $\mathcal{L}_1$ and $\mathcal{L}_2$, the shifted Lagrangian $\mathcal{L}_d$ would in general acquire linear and bilinear interaction terms $\mathcal{L}_{1d}$ and $\mathcal{L}_{2d}$ from $\mathcal{L}_3$ and/or $\mathcal{L}_4$ terms. (d) The quadrilinear interaction term $\mathcal{L}_4$ of the original Lagrangian $\mathcal{L}$ may contribute to the linear, bilinear, and/or trilinear terms $\mathcal{L}_{1d}$, $\mathcal{L}_{2d}$, and $\mathcal{L}_{3d}$. (e) The quadrilinear interaction terms of $\mathcal{L}$ and $\mathcal{L}_d$ are the same. Consequently, the shifted Lagrangian $\mathcal{L}_d$ may have linear, bilinear, and trilinear interaction terms, though they are absent in the original Lagrangian $\mathcal{L}$!

2.5. Inherited and Explicitly Broken Symmetries of Lagrangians

Suppose that the original Lagrangian $\mathcal{L}$ exhibits a certain symmetry, say

$$\mathbf{S}^{-1}\mathcal{L}(\mathbf{S}\Psi, \nabla\mathbf{S}\Psi) = \mathcal{L}(\Psi, \nabla\Psi) \qquad \forall\ \Psi \tag{2.28}$$

where $\mathbf{S}$ is a linear invertible transformation of states $\Psi(x, p)$ and of scalar fields of x such that for any two states Ψ_1 and Ψ_2,

$$\begin{aligned}
\mathbf{S}(c_1\Psi_1 + c_2\Psi_2) &= c_1\mathbf{S}\Psi_1 + c_2\mathbf{S}\Psi_2 \qquad \forall\ c_1, c_2 \in \mathrm{IR} \\
\langle \mathbf{S}\Psi_1 | \mathbf{S}\Psi_2 \rangle_p &= \mathbf{S}\langle \Psi_1 | \Psi_2 \rangle_p \\
p\cdot\nabla\mathbf{S}\Psi_1 &= \mathbf{S}p\cdot\nabla\Psi_1
\end{aligned} \tag{2.29}$$

Note that (2.28) with (2.29) is equivalent to

$$\mathbf{S}^{-1}\mathcal{L}_n(\mathbf{S}\Psi_1, \ldots, \mathbf{S}\Psi_n) = \mathcal{L}_n(\Psi_1, \ldots, \Psi_n), \qquad n = 1, 2, 3, 4 \tag{2.30}$$

by (2.1), (2.12), (2.7), (2.14), and (2.17). When Ψ is a solution to the Euler–Lagrange equations (2.20), then one can verify that also $\mathbf{S}\Psi$ is a solution to the same Euler–Lagrange equations (2.20). In particular, if $\Psi(x, p)$ is a solution to (2.20), then for any Lorentz transformation Λ and time-space shift $a \in \mathrm{IR}^{1,3}$ also the state $(\Lambda\Psi)(\Lambda^{-1}x + a, \Lambda^{-1}p)$ is a solution to (2.20), by (2.18)–(2.19). Further, if Ψ_d is a solution to the Euler–Lagrange equations of $\mathcal{L}_d$, then $\mathbf{S}\Psi_r + \mathbf{S}\Psi_d$ is a solution to the original Euler–Lagrange equations (2.20).

When the original Lagrangian $\mathcal{L}$ exhibits a certain symmetry (2.28), then it is of interest to know whether the shifted Lagrangian $\mathcal{L}_d$ derived from it exhibits the same symmetry with respect to the independent dynamical variable Ψ_d, or if this symmetry is explicitly broken. In general the symmetry will be explicitly broken; e.g., if $\mathcal{L}_1 = 0$ and $\mathcal{L}_3 = 0$, then $\mathcal{L}$ is invariant under the substitution of Ψ by $-\Psi$, but $\mathcal{L}_d$ will be invariant under the substitution of Ψ_d by $-\Psi_d$ only in particular cases. However, one can show that

$$\mathbf{S}^{-1}\mathcal{L}_d(\mathbf{S}\Psi_d, \nabla\mathbf{S}\Psi_d; \mathbf{S}\Psi_r) = \mathcal{L}_d(\Psi_d, \nabla\Psi_d; \Psi_r) \qquad \forall\, \Psi_d, \Psi_r \tag{2.31}$$

by (2.26)–(2.30), (2.12), (2.7), and (2.14). Thus, if Ψ_d is a solution to the Euler–Lagrange equations of the shifted Lagrangian $\mathcal{L}_d(\Psi_d, \nabla\Psi_d; \Psi_r)$ for a given reference state Ψ_r, then $\mathbf{S}\Psi_d$ is a solution to the Euler–Lagrange equations for Ψ_d of the Lagrangian $\mathcal{L}_d(\Psi, \nabla\Psi_d; \mathbf{S}\Psi_r)$. When the reference state Ψ_r is such that

$$\mathcal{L}_{id}(\Psi_d; \mathbf{S}\Psi_r) = \mathcal{L}_{id}(\Psi_d; \Psi_r) \qquad \forall\, \Psi_d \tag{2.32}$$

then the shifted Lagrangian $\mathcal{L}_d(\Psi_d, \nabla\Psi_d; \Psi_r)$ of the independent dynamical variable Ψ_d inherits the symmetry (2.28) of $\mathcal{L}$, by (2.26) and (2.31). In particular, we have (2.32) when the reference state Ψ_r is invariant under symmetry $\mathbf{S}$, though $\mathbf{S}\Psi_r = \Psi_r$ is not a necessary condition for (2.32). For example, the state $\Psi_r \equiv \psi \exp(ip\cdot x)$, ψ being a constant bispinor, is neither invariant under time-space shifts nor under Lorentz transformations Λ, but $\mathcal{L}_{id}(\Psi_d; \Psi_r)$ is when it depends on the reference state Ψ_r only through the scalar product $\overline{\Psi}_r\Psi_r$.

2.6. The Basic Lagrangian

In the Euler–Lagrange equations (2.20), the terms $\mathcal{L}_1(\Psi')$ and $2\mathcal{L}_2(\Psi', \Psi)$ describe the independent sources and interactions with an underlying medium whose properties do not depend on $\Psi(x, p)$, respectively. So it makes physical sense to assume that these two terms are absent from the Euler–Lagrange equations that give a complete description of the physical world, so that, were it not for the self-interaction terms $\mathscr{L}_3$ and $\mathscr{L}_4$, the state of the physical world would be a free field $\Psi_f(x, p)$. Thence, we put forward the hypothesis that in the case of the transport-theoretic Lagrangian $\mathscr{L}_f + \mathscr{L}_i$ that gives a complete description of the basic physical phenomena,

$$\mathscr{L}_1(\Psi) \equiv 0 \qquad \text{and} \qquad \mathscr{L}_2(\Psi, \Psi) \equiv 0 \tag{2.33}$$

i.e., we hypothesize that all physical phenomena are due solely to the free streaming and self-interaction described by $\mathscr{L}_f$ and $\mathscr{L}_3 + \mathscr{L}_4$, respectively. According to hypothesis (2.33), a Lagrangian density with linear and/or bilinear interaction terms can be interpreted in view of (2.26)–(2.27) as a shifted Lagrangian $\mathscr{L}_d$ whose linear and/or bilinear terms originate from the interaction of the state Ψ_d with some reference state Ψ_r.

2.7. Ground State

On assuming (2.33), it is physically convenient to generate the bilinear term $\mathscr{L}_{2d}$ (if needed) by using as a reference state $\Psi_r(x, p)$ a ground state, i.e., a particular reference state Ψ_r such that:

(i) It is invariant under Lorentz transformations and under time-space shifts, i.e., $(\Lambda\Psi_r)(\Lambda^{-1}x + a, \Lambda^{-1}p) = \Psi_r(x, p)$ for all Λ and for all $a \in \mathrm{IR}^{1,3}$. Thence, a ground state does not depend on the time-space variable x, say

$$\Psi_r(x, p) = \Psi_g(p) \qquad \text{and} \qquad (\Lambda\Psi_g)(\Lambda^{-1}p) = \Psi_g(p) \tag{2.34}$$

i.e., a ground state is a free state. As a consequence, the shifted Lagrangian $\mathscr{L}_d(\Psi_d, \nabla\Psi_d; \Psi_g)$ is invariant under Lorentz transformations and under time-space shifts, by (2.18), (2.19), and (2.32).

(ii) It is a solution to the Euler–Lagrange equations (2.20), i.e.,

$$3\mathscr{L}_3(\Psi', \Psi_g, \Psi_g) + 4\mathscr{L}_4(\Psi', \Psi_g, \Psi_g, \Psi_g) = 0 \qquad \forall\, \Psi' \tag{2.35}$$

by (2.21), (2.33), and (2.34). Thence, by (2.27), the linear interaction term $\mathscr{L}_{1d}$ is absent from the shifted Lagrangian $\mathscr{L}_d(\Psi_d, \nabla\Psi_d; \Psi_g)$, i.e.,

$$\mathscr{L}_{1d}(\Psi_d; \Psi_g) = 0 \qquad \forall\, \Psi_d \tag{2.36}$$

so that it is stationary in the following sense: $\mathscr{L}_d(\varepsilon\Psi_d, \nabla\varepsilon\Psi_d; \Psi_g) = O(\varepsilon^2)$ as $\varepsilon \to 0$.

Requirement (2.34) that the ground state $\Psi_g(p)$ is invariant under proper orthochronous Lorentz transformations Λ implies:

(i) When the states $\Psi(x, p)$ are complex (real) relativistic-scalar fields, then

$$\Psi_{g0}(p) \equiv f(p \cdot p) \tag{2.37}$$

is a ground state for any complex (real) function $f(y)$ of $y \in$ IR such that (2.35) holds.

(ii) When the states $\Psi(x, p)$ are spinor fields, then it seems that only the zero ground state $\Psi_g(p) = 0$ can satisfy (2.34).

(iii) When the states $\Psi(x, p)$ are complex (real) four-vector fields, then

$$\Psi_{g1}(p) \equiv f(p \cdot p)p \tag{2.38}$$

is a ground state for any complex (real) function $f(y)$ of $y \in$ IR such that $\Psi_{g1}(p)$ is a solution to (2.35). Thus, in contradistinction to conventional quantum field theories, within the transport-theoretic framework not only scalar bosons, but also four-vector bosons may have a Lorentz-invariant nonzero ground state.

2.8. Quartic Vertices as the Starting Point

In the kinetic theory of gases, interactions are due to collisions without absorption between two or more particles. This fact could suggest that in quantum field theories quartic vertices are actually basic and, therefore, any three-leg vertex is actually a four-leg vertex with one of its legs standing for interaction with a constant Lorentz-invariant bosonic ground state and therefore suppressed and taken into account by the coupling constant of the resulting three-leg vertex. If so, the interaction part $\mathscr{L}_i$ of the basic transport-theoretic Lagrangian $\mathscr{L}_f + \mathscr{L}_i$ underlying quantum field theories has only a quadrilinear interaction term $\mathscr{L}_4(\Psi, \Psi, \Psi, \Psi)$ and

$$\mathscr{L}_1(\Psi) \equiv 0, \qquad \mathscr{L}_2(\Psi, \Psi) \equiv 0, \qquad \mathscr{L}_3(\Psi, \Psi, \Psi) \equiv 0 \tag{2.39}$$

In such a case, the quadrilinear term $\mathscr{L}_4$ is the origin of any linear, bilinear, and trilinear interaction terms $\mathscr{L}_{1d}$, $\mathscr{L}_{2d}$, and $\mathscr{L}_{3d}$ the shifted Lagrangian $\mathscr{L}_f + \mathscr{L}_{id}$ may have.

If for any complex constants c_1, c_2, c_3, and c_4,

$$\mathscr{L}_4(c_1\Psi, c_2\Psi, c_3\Psi, c_4\Psi) = \tfrac{1}{3}\Re(c_1^*c_2^*c_3c_4 + c_1^*c_2c_3^*c_4 + c_1^*c_2c_3c_4^*)\mathscr{L}_4(\Psi, \Psi, \Psi, \Psi) \tag{2.40}$$

then the Lagrangian $\mathscr{L} = \mathscr{L}_f + \mathscr{L}_4$ is invariant under a phase change, i.e.,

$$\mathscr{L}(e^{i\alpha}\Psi, \nabla e^{i\alpha}\Psi) = \mathscr{L}(\Psi, \nabla\Psi) \qquad \forall\, \alpha \in \mathrm{IR} \tag{2.41}$$

and $\langle\Psi|p\Psi\rangle_p$ is a conserved current when Ψ is a solution to the Euler–Lagrange equations (2.20), by (2.40) and (2.23). In general, a modified Lagrangian density $\mathscr{L}_d$ will not inherit the symmetry (2.41); it will be hidden in the relation

$$\mathscr{L}_d(e^{i\alpha}\Psi_{d,}\ \nabla e^{i\alpha}\Psi_d;\ e^{i\alpha}\Psi_r) = \mathscr{L}_d(\Psi_d,\ \nabla\Psi_d;\ \Psi_r) \qquad \forall\ \alpha \in \mathrm{IR} \quad (2.42)$$

by (2.31). Thus one can use the proposed hypothesis (2.39)–(2.40) to give a physical interpretation of the invariance of Lagrangians $\mathscr{L}_{QED}$ and $\mathscr{L}_{SQED}$ of quantum electrodynamics (3.1) and of scalar electrodynamics (3.16) under the substitution of A by $e^{i\alpha}A$, e by $e^{-i\alpha}e$, ϕ by $e^{i\alpha}\phi$, and $\psi(x)$ by $e^{i\alpha}\psi(x)$, $\alpha \in \mathrm{IR}$; also, the Lagrangian of the standard model can be interpreted so that it exhibits an analogous invariance that can be interpreted as a broken symmetry (2.41).

3. QUANTUM FIELD THEORIES

3.1. Quantum Electrodynamics

Quantum electrodynamics of charged spin-1/2 particles is characterized by the Lagrangian density

$$\mathscr{L}_{QED} \equiv -\tfrac{1}{4}|\nabla \otimes A - (\nabla \otimes A)^t|^2 - \tfrac{\lambda}{2}|\nabla\cdot A|^2 + \mathscr{R}\{\overline{\psi}[i\gamma^\mu\nabla_\mu - e\gamma^\mu A_\mu - m]\psi\} \qquad (3.1)$$

in which $\psi(x)$ and $A(x)$ are complex, bispinor and four-vector fields of the time-space variable $x \in \mathrm{IR}^{1,3}$, $|M|^2 \equiv M^*_{\alpha\beta}M^{\alpha\beta}$ for any second-order four-tensor M, $\otimes$ denotes the dyadic product of two four-vectors, the superscript t designates the transposed tensor, $\overline{\psi} \equiv \psi^\dagger\gamma^0$, the dagger denotes the adjoint (transposed and complex conjugated) spinor, γ^μ are the Dirac matrices, and λ, e, and m are real constants. The Lagrangian $\mathscr{L}_{QED}$ is a real relativistic-scalar field of x; for real fields $A(x)$, it equals the usual Lagrangian of quantum electrodynamics up to a divergence term (Itzykson and Zuber, 1987). We are going to show how one can derive $\mathscr{L}_{QED}$ from a particular transport-theoretic Lagrangian of Section 2 shifted with respect to a ground state, so that $\mathscr{L}_{1d} = 0$.

To this end we assume that the state

$$\Psi_d(x, p) = \begin{pmatrix} \Psi_{1/2}(x, p) \\ \Psi_1(x, p) \end{pmatrix} \qquad (3.2)$$

where $\Psi_{1/2}$ and Ψ_1 are complex, left-handed-spinor and four-vector fields of x, $p \in \mathrm{IR}^{1,3}$, respectively. In such a case, the ground state $\Psi_g(x, p) = (0, \Psi_{g1}(p))^t$. We take the scalar product

$$\begin{aligned} \langle\Psi_d|\Psi'_{d}\rangle_p &\equiv \langle\Psi_{1/2}|\Psi'_{1/2}\rangle_{1/2} + \langle\Psi_1|\Psi'_1\rangle_1 \\ \langle\Psi_{1/2}|\Psi'_{1/2}\rangle_{1/2} &\equiv \mathbf{I}_p\overline{\Psi}^\dagger_{1/2}(x, p)\Psi'_{1/2}(x, p) \\ \langle\Psi_1|\Psi'_1\rangle_1 &\equiv \mathbf{I}_p\overline{\Psi}^*_1(x, p)\cdot\Psi_1(x, p) \end{aligned} \qquad (3.3)$$

where for complex values of the four-momentum variable $p = (p^0, \mathbf{p})$,

$$\overline{\Psi}_{1/2}(x, p) \equiv i[p^{0*} + \mathbf{p}^*\cdot\boldsymbol{\sigma}]\Psi_{1/2}(x, -p^*)$$
$$\overline{\Psi}_1(x, p) \equiv \Psi_1(x, -p^*) \tag{3.4}$$

with

$$p^2 + \mathbf{p}\cdot\boldsymbol{\sigma} \equiv \begin{pmatrix} p^0 + p^3 & p^1 - ip^2 \\ p^1 + ip^2 & p^0 - p^3 \end{pmatrix} \tag{3.5}$$

and the momentum total

$$\mathbf{I}_p F(p) \equiv \lim_{R\to\infty} \int_{-R}^{R} dy \int_{|p|^2 \leq R^2 - y^2} F(iy, \mathbf{p})\, d^3\mathbf{p} \tag{3.6}$$

for any function $F(p) = F(p^0, \mathbf{p})$ such that the limit (3.6) exists, the values of $\Psi_d(x, p)$ and $\Psi_d'(x, p)$ for complex values of the four-momentum variable p being defined by analytic continuation of $\Psi_d(x, p)$ and $\Psi_d'(x, p)$, $p \in \mathrm{IR}^{1,3}$. The definition (3.6) of the integral over the independent variable p is suggested by the standard, covariant, Euclidean definition of a four-momentum integral through the Wick rotation in momentum space, followed by symmetric integration. For a fairly general class of states $\Psi(x, p)$, one can show (Ribarič and Šušteršič, 1995) that the scalar product (3.3) has the properties (2.6)–(2.11).

The shifted transport-theoretic Lagrangian $\mathcal{L}_d$ depends on fields $\Psi_{1/2}(x, p)$ and $\Psi_1(x, p)$ of 4 + 4 independent variables (the components of x and p), whereas the quantum electrodynamic Lagrangian $\mathcal{L}_{QED}$ depends on fields $\psi(x)$ and $A(x)$ of only 4 independent variables. So the crucial question is how to achieve this transition from a transport-theoretic Lagrangian with states of eight independent variables to a Lagrangian with states depending on just four of them. Taking a clue from the kinetic theory of gases, where in the strong-scattering asymptote as the mean free path tends to zero the gas exhibits everywhere the same Maxwellian distribution of velocities, we define an analogous strong-scattering asymptote to accomplish this reduction of dimensionality as follows: We assume that the interaction Lagrangian $\mathcal{L}_{id}(\Psi_d; \Psi_g)$ depends on the gauge parameter λ in such a way that in the strong-scattering asymptote $\lambda \to 0$ it tends to infinity unless the state

$$\Psi_d(x, p) = \Psi_a(x, p) \equiv \begin{pmatrix} f_{1/2}(p\cdot p)[(-p\cdot p)^{1/2}\phi_L(x) + i(p^0 - \mathbf{p}\cdot\boldsymbol{\sigma})\phi_R(x)] \\ f_1(p\cdot p)A(x) + f_1'(p\cdot p)M(x)p \end{pmatrix} \tag{3.7}$$

where $\phi_L(x)$ and $\phi_R(x)$ are some complex, left- and right-handed spinor fields of x, respectively; $A(x)$ and $M(x)$ are some complex, four-vector and second-rank four-tensor fields of x, respectively; and $f_{1/2}(y), f_1(y)$, and $f_1'(y)$ are certain complex functions of $y \in \mathbb{R}$ (but not of λ) such that

$$c_{1/2} \equiv \tfrac{1}{4}\pi^2 \int_0^\infty r^3 |f_{1/2}(-r)|^2 \, dr > 0$$

$$c_1 \equiv \tfrac{1}{4}\pi^2 \int_0^\infty r^2 f_1^*(-r) f_1'(-r) \, dr \neq 0 \tag{3.8}$$

(For what follows, there is actually no mathematical need to assume that the parameter that tends to zero in the strong-scattering asymptote is the same as the gauge parameter λ in $\mathcal{L}_{\text{QED}}$, but it saves introducing an additional parameter.) Since, by (2.12) and (3.3)–(3.6), the free Lagrangian $\mathcal{L}_f(\Psi_d, \nabla\Psi_d)$ is defined so that it does not depend on λ, in the strong-scattering asymptote $\lambda \to 0$ the Lagrangian $\mathcal{L}_d(\Psi_d, \nabla\Psi_d; \Psi_g)$ and the corresponding action $I[\Psi]$ defined by (1.1) both tend to infinity unless the state $\Psi_d(x, p)$ is of the form (3.7). We assume, therefore, that in the strong-scattering asymptote $\lambda \to 0$, only those states $\Psi_d(x, p)$ that are given by (3.7) contribute to the path integral (1.2); i.e., we assume that in the strong-scattering asymptote $\lambda \to 0$, in the functional integral (1.2) we need take account only of the states $\Psi_d(x, p) = \Psi_a(x, p)$. If so, in the asymptote $\lambda \to 0$, the physical system considered is described by the asymptotic Lagrangian $\mathcal{L}_f(\Psi_a, \nabla\Psi_a) + \mathcal{L}_{id}(\Psi_a; \Psi_g)$, where:

(A) By (3.2)–(3.8), the free Lagrangian

$$\mathcal{L}_f(\Psi_a, \nabla\Psi_a) = \mathcal{R}\{c_1^*(M^*\nabla)\cdot A - c_1 A^*\cdot(\nabla M') + ic_{1/2}\bar{\psi}\gamma^\mu\nabla_\mu\psi\} \tag{3.9}$$

where $\psi(x)$ is the complex bispinor field such that in the chiral representation

$$\psi(x) = \begin{pmatrix} \phi_R(x) \\ \phi_L(x) \end{pmatrix} \tag{3.10}$$

(B) Assumption (3.7) and the properties (i)–(v) in Section 2.2 of the interaction terms imply that $\mathcal{L}_{2d}$ [$\mathcal{L}_{3d}$, $\mathcal{L}_4$] with $\Psi_d = \Psi_a$ is such a sum of the second- [third-, fourth-] order products of $\psi(x)$, $A(x)$, and $M(x)$ that is a real relativistic-scalar field of x. For our purpose we can assume, e.g., that

$$\begin{aligned} \mathcal{L}_{2d}(\Psi_a, \Psi_a; \Psi_g) &= -c_{1/2} m\bar{\psi}\psi - 2t_1|M|^2 - \tfrac{1}{2}t_1^2 c_{1/2}|c_1|^{-2}[|M|^2 \\ &\qquad + (\lambda + 1)|\text{Tr } M|^2] \\ \mathcal{L}_{3d}(\Psi_a, \Psi_a, \Psi_a; \Psi_g) &= -c_{1/2}\mathcal{R}\{e\bar{\psi}A_\mu\gamma^\mu\psi\}, \\ \mathcal{L}_{4d}(\Psi_a, \Psi_a, \Psi_a, \Psi_a) &= 0 \end{aligned} \tag{3.11}$$

Tr M is the trace of M and t_1 a real parameter different from zero.

If we assume that

$$t_1 M(x) = c_1^*(\nabla \otimes A)^t \tag{3.12}$$

then the asymptotic transport-theoretic Lagrangian $\mathcal{L}_f(\Psi_a, \nabla\Psi_a) + \mathcal{L}_{id}(\Psi_a; \Psi_g)$ as defined by (3.9)–(3.11) is equal to the quantum-electrodynamic Lagrangian $\mathcal{L}_{QED}$, we set out to derive, up to the factor $c_{1/2}$ and a divergence term.

The physical significance of assumption (3.12), needed to derive terms quadratic in $\nabla \otimes A$ such that are present in $\mathcal{L}_{QED}$, is open. Relation (3.12) may be given a transport-theoretic interpretation, e.g., in the following three cases:

(i) The four-vector component $\Psi_1(x, p)$ of any state $\Psi_d(x, p)$ must satisfy the covariant constraint

$$\langle \Psi_{g1}(p) | [p \cdot \nabla - f_c(p \cdot p)][a \cdot \Psi_1] b \rangle_1 = 0 \qquad \forall\, a, b \in \mathrm{IR}^{1,3} \tag{3.13}$$

where $f_c(y)$ is some real function of $y \in \mathrm{IR}$ such that

$$c_1^* \int_0^\infty r^2 f^*(-r) f_1'(-r) f_c(-r)\, dr = t_1 \int_0^\infty r^2 f^*(-r) f_1(-r)\, dr \tag{3.14}$$

for $\Psi_d = \Psi_a$, this constraint (3.13) reduces to (3.12).

(ii) The four-vector component $\Psi_1(x, p)$ of any state $\Psi_d(x, p)$ satisfies a covariant generalization of Fick's first law

$$\Psi_1(x, p) - \Psi_1(x, -p) = f_f(p \cdot p) p \cdot \nabla [\Psi_1(x, p) + \Psi_1(x, -p)] \tag{3.15}$$

where $f_f(y)$ is some real function of $y \in \mathrm{IR}$ (Ribarič and Šušteršič, 1987); for $\Psi_1(x, p) = f_1(p \cdot p) A(x) + f_1'(p \cdot p) M(x) p$, relation (3.15) is equivalent to (3.12) provided $t_1 f_f(y) f_1(y) = c_1^* f_1'(y)$.

(iii) Like the free Lagrangian $\mathcal{L}_f(\Psi, \nabla\Psi)$, the interaction Lagrangian $\mathcal{L}_{id}$ depends locally not only on the state Ψ_d but also on its time-space derivative $\nabla\Psi_d$. In such a case, it seems that one is allowed to assume (3.7) and (3.11) with $M(x)$ replaced by $c_1^* t_1^{-1} (\nabla \otimes A)^t$.

It does not seem possible to derive the quadratic, mass, and gauge terms of the quantum-electrodynamic Lagrangian $\mathcal{L}_{QED}$ if the shifted transport-theoretic Lagrangian $\mathcal{L}_f + \mathcal{L}_{id}$ does not have a bilinear interaction term $\mathcal{L}_{2d}(\Psi_d, \Psi_d; \Psi_g)$. If we adopt the hypothesis (2.33) that the basic transport-theoretic Lagrangian $\mathcal{L}_f + \mathcal{L}_i$ has no bilinear interaction term, then we have to conclude that the Lagrangian $\mathcal{L}_f + \mathcal{L}_{id}$, needed to obtain $\mathcal{L}_{QED}$ in the strong-scattering asymptote $\lambda \to 0$, is a result of some nonzero shift as defined by (2.24). Consequently:

(i) We can expect that some of the symmetries exhibited by the basic Lagrangian $\mathcal{L}_f + \mathcal{L}_i$ will not be inherited by the shifted Lagrangian $\mathcal{L}_f$ +

$\mathcal{L}_{id}$ and, therefore, in the quantum electrodynamic Lagrangian $\mathcal{L}_{QED}$, they will be explicitly broken.

(ii) Quantum fields $A(x)$ and $\psi(x)$ describe in the asymptote $\lambda \to 0$ the addition $\Psi_d(x, p)$ to the ground state $\Psi_g(x, p) = (0, \Psi_{g1})^t$.

(iii) Since by (3.11) the spinor field $\Psi_{1/2}(x, p)$ acquires mass through interaction with the nonzero ground state $\Psi_{g1}(p)$ of four-vector field $\Psi_1(x, p)$, it is the electromagnetic ground state $\Psi_{g1}(p)$ and the shift (2.24) of the Lagrangian $\mathcal{L}_f + \mathcal{L}_i$ that, in the transport-theoretic interpretation of quantum electrodynamics, play a role analogous to that of the Higgs mechanism in the standard model (Cheng and Li, 1992).

3.2. Scalar Electrodynamics

Scalar electrodynamics, i.e., quantum electrodynamics of charged spin-zero particles, may be characterized by the Lagrangian

$$\begin{aligned}\mathcal{L}_{SQED} \equiv {} & -\tfrac{1}{4}|\nabla \otimes A - (\nabla \otimes A)^t|^2 - \tfrac{\lambda}{2}|\nabla \cdot A|^2 \\ & + [\nabla\phi + ieA\phi]^* \cdot [\nabla\phi + ieA\phi] - m^2\phi^*\phi - \tfrac{1}{4}g(\phi^*\phi)^2\end{aligned} \tag{3.16}$$

in which $\phi(x)$ and $A(x)$ are complex, scalar and four-vector fields of the time-space variable $x \in \mathrm{IR}^{1,3}$, and e, m, g, and λ are real constants. The Lagrangian $\mathcal{L}_{SQED}$ is a real relativistic-scalar field of x; for real fields $A(x)$, it equals the conventional Lagrangian of scalar electrodynamics (Itzykson and Zuber, 1987).

Let us sketch how one can derive $\mathcal{L}_{SQED}$ from a transport-theoretic Lagrangian $\mathcal{L}_f + \mathcal{L}_{id}$ of Section 2, if we assume that

$$\Psi_d(x, p) = \begin{pmatrix}\Psi_0(x, p)\\ \Psi_1(x, p)\end{pmatrix} \tag{3.17}$$

where Ψ_0 and Ψ_1 are complex, scalar and four-vector fields of $x, p \in \mathrm{IR}^{1,3}$, respectively; here, the ground state is $\Psi_g(x, p) = (\Psi_{g0}(p), \Psi_{g1}(p))^t$, and the scalar product is

$$\langle\Psi_d|\Psi_d'\rangle_p \equiv \langle e^0\Psi_0|e^0\Psi_0'\rangle_1 + \langle\Psi_1|\Psi_1'\rangle_1 \qquad \text{with} \quad e^0 \equiv (1, 0, 0, 0) \tag{3.18}$$

We will consider a case where interaction terms $\mathcal{L}_{2d}(\Psi_d, \Psi_d; \Psi_g)$, $\mathcal{L}_{3d}(\Psi_d, \Psi_d, \Psi_d; \Psi_g)$, and $\mathcal{L}_4(\Psi_d, \Psi_d, \Psi_d, \Psi_d)$ depend on the gauge parameter λ in such a way that in the asymptote $\lambda \to 0$ the action (1.1) tends to infinity unless

$$\Psi_d(x, p) = \Psi_a(x, p) \equiv \begin{pmatrix}f_0(p\cdot p)\phi(x) + f_0'(p\cdot p)b(x)\cdot p\\ f_1(p\cdot p)A(x) + f_1'(p\cdot p)M(x)p\end{pmatrix} \tag{3.19}$$

where $\phi(x)$, $b(x)$, $A(x)$, and $M(x)$ are some complex, scalar, four-vector, four-vector, and second-rank four-tensor fields of x, respectively; and $f_0(y)$,

$f_0'(y)$, $f_1(y)$, and $f_1'(y)$, are certain complex functions of $y \in \mathbb{R}$, independent of λ and such that

$$c_j \equiv \tfrac{1}{4}\pi^2 \int_0^\infty r^2 f_j^*(-r) f_j'(-r) dr \neq 0, \qquad j = 0, 1 \tag{3.20}$$

By (3.17)–(3.20), the free Lagrangian density $\mathcal{L}_f$ does not depend on λ, and

$$\mathcal{L}_f(\Psi_a, \nabla\Psi_a) = \mathfrak{R}\{c_0^* b^* \cdot \nabla\phi - c_0 \phi^* \nabla \cdot b + c_1^*(M^*\nabla) \cdot A - c_1 A^* \cdot (\nabla M^t)\} \tag{3.21}$$

Suppose that, e.g.,

$$\begin{aligned} \mathcal{L}_{2d}(\Psi_a, \Psi_a; \Psi_g) &= 2(1 + \lambda)t_1 |\mathrm{Tr}\, M|^2 - m^2 b^* b \\ \mathcal{L}_{3d}(\Psi_a, \Psi_a, \Psi_a; \Psi_g) &= 2t_0 \mathfrak{R}\{iec_0^{-1} \phi b^* \cdot A) \\ \mathcal{L}_4(\Psi_a, \Psi_a, \Psi_a, \Psi_a) &= e^2 |\phi A|^2 - \tfrac{1}{4} g [\phi^* \phi]^2 \end{aligned} \tag{3.22}$$

where t_0 and t_1 are two real parameters; and let

$$t_1 M(x) = c_1^* (\nabla \otimes A)^t \qquad \text{and} \qquad t_0 b(x) = c_0^* \nabla \phi(x) \tag{3.23}$$

In such a case, the asymptotic transport-theoretic Lagrangian $\mathcal{L}_f(\Psi_a, \nabla\Psi_a) + \mathcal{L}_{id}(\Psi_a; \Psi_g)$ with $t_0 = 2|c_0|^2$ and $t_1 = -4|c_1|^2$ is equal to $\mathcal{L}_{SQED}$ up to a divergence term. There are transport-theoretic interpretations of constraints (3.23) analogous to those of (3.12). Due to the second constraint in (3.23), we were able to obtain the derivative coupling in $\mathcal{L}_{SQED}$ without having to assume that a derivative coupling is already present in the interaction part $\mathcal{L}_{id}$ of the transport-theoretic Lagrangian.

To obtain transport-theoretic derivations of $\mathcal{L}_{QED}$ and of $\mathcal{L}_{SQED}$ we have given explicit expressions for the free part $\mathcal{L}_f$ of the transport-theoretic Lagrangians needed, but only a few qualitative properties (2.14)–(2.17) of the interaction part $\mathcal{L}_i$. It would be of interest to identify further properties that interaction parts $\mathcal{L}_i$ of physically relevant transport-theoretic Lagrangians have to display.

4. CONCLUDING REMARKS

Presuming the Lagrangians of fundamental interactions in quantum field theories to be just asymptotic approximations to some underlying Lagrangian, we have considered the possibility that such an underlying Lagrangian is of the transport-theoretic kind, with the states $\Psi(x, p)$ depending on eight independent variables (four components of the conventional space-time variable $x \in \mathrm{IR}^{1,3}$ and four components of the four-momentum variable $p \in \mathrm{IR}^{1,3}$). In Section 3, we introduced the concept of the strong-scattering asymptote, where the gauge parameter $\lambda \to 0$ and only particular states $\Psi(x, p)$, which are products of the usual quantum mechanical fields of x and of certain functions of the four-momentum variable p, contribute to the path integral (1.2). We have also shown how one can interpret Lagrangians of quantum and scalar electrodynamics as describing some underlying transport-theoretic Lagrangians in the strong-scattering asymptote; also, the Lagrangians of non-Abelian gauge theories can be interpreted this way. According to this interpretation, by using the gauge parameter, regularizations, and renormalizations in relativistic quantum field theories one is in effect simulating a postponement of a certain asymptotic approximation so as to derive proper, finite, asymptotic results implied by the underlying transport-theoretic Lagrangian.

Within the transport-theoretic framework, a four-vector field may have a nonzero ground state (2.38) that does not depend on the time-space variable x and is an eigenvector of the Lorentz transformations. According to the given transport-theoretic interpretation of quantum electrodynamics, fermions acquire their mass through interaction with the electromagnetic ground state in the strong-scattering asymptote. Thus, within the transport-theoretic framework, interactions with nonzero ground states of vector bosons present an alternative to the Higgs mechanism of the standard model. So, if it turns out that we will give the scalar Higgs boson up since there will be no satisfactory experimental upper bound on its mass forthcoming, it might be worth considering whether massless, vector-boson and fermion fields acquire mass by interaction with the ground states of vector bosons (fermions being absent from the ground state since spinor fields do not have a nonzero ground state).

On the analogy of the kinetic theory of gases, we cannot expect that the concept of mass had any physical meaning immediately after the big bang when the state $\Psi(x, p)$ was changing rapidly as a function of x and therefore could not be approximated by a sum $\Psi_g(p) + \Psi_a(x, p)$ of the ground and asymptotic states needed to transform the transport-theoretic Lagrangian into the conventional Lagrangian of the standard model that defines masses.

Regarding the relation between gravitation and the transport-theoretic hypothesis that may be interpreted as attributing physical phenomena to an underlying motion and interaction of pointlike entities, we see two open questions:

(i) Can we apply the transport hypothesis to gravitation?

(ii) Does the ground state or/and the said hypothetical pointlike entities exert some gravitational interaction? Have they anything to do with the so-called cold dark matter proposed as a way to explain the missing mass problem?

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